Problem 1 - Reputation Dynamics: Consider a reputation value $0 \leq r_k \leq 1$ (0 meaning malicious, 1 meaning cooperative) evaluating past node behavior over a sequence of interactions with $k$ as a time index. After the $k^{th}$ interaction, an observer of $n$ updates its metric to $r_k(n)$ as a function of the previous value $r_{k-1}(n)$. Consider the following update methods:

(i) Additive reward ($\epsilon_1 > 0$), additive penalty ($\epsilon_0 > 0$):

$$r_k(n) = \begin{cases} 
\min (r_{k-1}(n) + \epsilon_1, 1) & \text{if } n \text{ cooperates} \\
\max (r_{k-1}(n) - \epsilon_0, 0) & \text{else.}
\end{cases}$$

(ii) Additive reward ($\epsilon_1 > 0$), multiplicative penalty ($0 \leq \delta_0 < 1$):

$$r_k(n) = \begin{cases} 
\min (r_{k-1}(n) + \epsilon_1, 1) & \text{if } n \text{ cooperates} \\
\delta_0 r_{k-1}(n) & \text{else.}
\end{cases}$$

(iii) Multiplicative reward ($0 \leq \delta_1 < 1$), multiplicative penalty ($0 \leq \delta_0 < 1$):

$$r_k(n) = \begin{cases} 
1 - \delta_1 (1 - r_{k-1}(n)) & \text{if } n \text{ cooperates} \\
\delta_0 r_{k-1}(n) & \text{else.}
\end{cases}$$

(a) (8 points) Discuss some of the positive and negative aspects of these different types of reputation dynamics. Suggest another method and compare it to the above.

(b) (7 points) Using software of your choice, generate two random sequences of 1000 “interactions” each, the first with about 50 malicious interactions and the second with about 950 malicious interactions. For each sequence, plot the reputation metrics above (with $r_0(n) = 1/2$) for several $\epsilon_i$ and $\delta_i$ values and discuss the additional effects that these parameters have.